

Some results on the energy transfer from  
the wind to the water during BOSEX 77

by

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Abstract

During BOSEX 77 a sharp drop of the temperature as well as considerable deepening of the mixed layer was observed during the passage of a storm. These observations together with current meter data and wind data are used to study the efficiency of the mechanical mixing induced by the wind relative to the totally available wind energy at about 10 m above the sea surface.

Problem statement

Vertical mixing in the sea is generally very weak due to the dominating stable stratification (see e.g. Kullenberg 1974). For the surface layer the mechanical energy input from the wind, directly or via the waves, is often crucial for generating vertical mixing during the warm seasons. It has been established that only a small fraction of the energy input from the wind is consumed for vertical mixing (e.g. Turner 1959; Denman and Miyake 1973; Kullenberg 1976). This is also in agreement with laboratory experiments (e.g. Turner 1973). It seems reasonable to expect that the strongest wind action occurs during storms and passages of meteorological fronts. The purpose of this note is to investigate the efficiency of wind-generated mixing during the passage of a storm using observations obtained during the Baltic Open Sea Experiment in September 1977, referred to as BOSEX 77. The vertical mixing during weak wind conditions was also investigated during BOSEX 77 by means of dye diffusion experiments. Thus conditions under fairly extreme situations can be compared.

Theoretical background

In the mixed layer below the wave zone horizontal homogeneity of the fluctuating fields may be assumed, implying that the mechanical energy equation can be written in the form

$$\frac{\partial}{\partial t} \left( \frac{\overline{E'^2}}{2} \right) + \frac{\partial}{\partial z} \left( w' \left( \frac{p'}{\rho_0} + \frac{E'^2}{2} \right) \right) + \overline{w' \vec{v}' \cdot \frac{\partial \vec{v}'}{\partial z}} + g \frac{\overline{\rho' w'}}{\rho_0} + \epsilon = 0 \quad (1)$$

Here  $\frac{1}{2} \overline{E'^2}$  is the fluctuating kinetic energy per unit mass,  $w'$ ,  $p'$  and the  $\rho'$  fluctuating parts of the vertical velocity, pressure and density, respectively,  $\rho_0$  is the mean density,  $\vec{v}'$  is the horizontal velocity vector with the fluctuating part  $\vec{v}'$ ,  $g$  is acceleration of gravity and  $\epsilon$  is the

rate of energy dissipation per unit mass. Absorption of solar radiation has been neglected.

The second term is the kinetic energy flux caused by vertical divergence, the third term gives the production of turbulent energy by interactions between the mean shear and the fluctuating velocity fields, and the fourth term gives the change of potential energy. Using the K-theory approximation this term may be written in the form

$$g \cdot \frac{\overline{\rho'w'}}{\rho_0} = \frac{-g}{\rho_0} K \frac{d\rho}{dz} = K N^2 \quad (2a)$$

where  $N$  is the Brunt-Väisälä frequency.

Similarly the production term may be written in the form

$$\overline{u'w'} \frac{dU}{dz} = K_m \cdot \left( \frac{dU}{dz} \right)^2 \quad (2b)$$

where  $K_m$  is the turbulent momentum transfer coefficient, and  $u'$ ,  $U$  are the fluctuating and mean velocities, respectively, in the x-direction, assuming mean motion in this direction only.

The time required for redistribution of turbulent kinetic energy in the mixed layer is small compared to the time over which the energy input from the wind changes, implying that a steady state may be considered.

Niiler (1975) suggested that away from boundaries a balance exist between the divergence, the buoyancy and the dissipation term. However, at the base of the mixed layer some production of turbulent energy will take place due to the shear there. A balance which is often assumed is between the production, buoyancy and dissipation terms, for instance analysed by Townsend (1958) in the atmospheric boundary layer.

Deepening of the mixed layer in the stably stratified case occurs through entrainment of fluid from the lower layer into the mixed layer due to well developed turbulence in the upper layer (see e.g. Phillips 1977, Turner 1973). The entrainment process appears to be dominating during high (storm) wind speeds. By means of a simple energy argument (e.g. Turner 1973) one finds that

$$u_e = c \cdot u_{*x} \cdot Ri_{*x}^{-1} \quad (3)$$

where  $u_e$  and  $u_{*x}$  are the entrainment and friction velocities, respectively,  $Ri_{*x}$  is overall Richardson number and  $c$  a constant. Kato and Phillips (1969) found  $c \approx 2.5$  from laboratory experiments. The rate of deepening of the mixed layer varies with time, in the early stage being proportional to  $t^{1/2}$ , after about one inertial period decreasing to  $t^{1/3}$ . These regimes have been found theoretically and to some extent confirmed experimentally (Kraus and Turner 1967, Pollard, Rhines and Thompson 1973, de Scoeke and Rhines 1976, Kundu 1980). Most models, however, has as initial conditions a fluid at rest with a constant slope density profile and a well-defined onset time of the wind stress or stirring, which also is treated as constant. In the sea these conditions are never met, and mostly the initial conditions are not even known.

It is clearly of interest to analyse individual storm cases in order to obtain more field information on the entrainment process and the efficiency of strong winds to generate vertical mixing in stably stratified conditions. Here we study the change of potential energy relative to

the mechanical energy available in the wind field at about 10 m above the sea surface. The rate of work per unit area by the wind stress at the 10 m level is written in the form

$$E_{10} = \tau_o W_{10} = c_d \rho_a W_{10}^3 \quad (4)$$

The energy input per unit time and area from the wind can be expressed as

$$E = k \tau_o W_{10} = k_1 u_* \tau_o \quad (5)$$

Here  $W_{10}$  is the wind speed,  $\tau_o$  the wind stress,  $c_d$  the drag coefficient,  $\rho_a$  the density of the air,  $k$  the windfactor, and  $k_1$  a numerical constant.

The potential energy per unit area of the water column is defined as

$$E_{\text{pot}} = g \int_{-D}^0 (\rho - \bar{\rho}) (z + D) dz \quad (6)$$

where  $\bar{\rho}$  is the mean reference density, and  $D$  the water depth. The value of  $E_{\text{pot}}$  is negative in stable conditions and zero for the totally mixed water column. The change of potential energy and the work by the wind over a period of time  $T$  are found by integration of (6) and (4), respectively, and the ratio  $R_1$  becomes

$$R_1 = \frac{\frac{g}{T} \int_{-D}^0 [\rho(t_1) - \bar{\rho}(t_1)] (z+D) dz - \frac{g}{T} \int_{-D}^0 [\rho(t_2) - \bar{\rho}(t_2)] (z+D) dz}{\frac{1}{T} \rho_a \int_{t_1}^{t_2} c_d W_{10}^3 dt} = \quad (7)$$

$$= \frac{\frac{g}{T} \int_{t_1}^{t_2} \int_{-D}^0 \frac{\partial}{\partial t} [\rho(t') - \bar{\rho}(t')] \cdot (z + D) dz dt}{\frac{1}{T} \rho_a \int_{t_1}^{t_2} c_d W_{10}^3 dt}$$

The aim is to determine this ratio from the observations.

The local flux Richardson number  $R_f$  is defined as the ratio between the potential energy change generated by the mechanical mixing and the total turbulent energy available, i.e.

$$R_f = \frac{\Delta E_{\text{pot}}}{E_T} = \frac{KN^2}{K_m \left(\frac{dU}{dz}\right)^2} = \frac{K}{K_m} \cdot Ri \quad (8)$$

where  $E_T$  is the total turbulent energy available and  $Ri$  is the gradient Richardson number. This formulation assumes that the only production of turbulence occurs through the interactions between the mean shear and the fluctuating velocities. In the present case we may assume that the energy input from the wind  $E$  is the source of turbulent energy so that  $E = E_T$ . Taking the mean of  $\Delta E_{\text{pot}}$  and  $E_t$  over some time period and assuming that the wind factor  $k$  is constant over this period we find

$$R_f \simeq R_1/k$$

where  $R_{f_c}$  is the critical flux Richardson number, i.e. the maximum value of  $R_f$ .

#### Data and results

During BOSEX 77 three Danish current meter stations were operated over the period 7 to 19 September (Table 1) which will be used for the present study. The time period analysed here is from 2 a.m. on 12 September to 2 a.m. on 14 September during which a storm passed the area (Table 2). The current speeds at 20, 30 and 40 m nominal depths of station 611 are shown in Figs. 1 - 3, with a direction record in Fig. 4. The recording interval was 10 minutes.

The temperature record (Fig. 5) shows a relatively rapid decrease of temperature in the upper layer and a rise in the intermediate layer, demonstrating the combined effect of atmospheric cooling, entrainment and diffusion in the water. Real time temperature was used in the calculations. However, horizontal advection changed the salinity field with the inertial period (Fig. 5) especially in the lower layer. For this reason the mean salinity during the period was used in the calculations. The change in potential energy was calculated over different periods  $T$ , ranging from 8 to 48 hours. The wind observations from R/V Alkor were used (Table 2) and, separately, the wind speeds observed by R/V Poseidon. These were considerably higher than those from R/V Alkor, implying an increase of  $E_{10}$  by a factor of 2.5. The values of the ratio  $R_1$  based on the Alkor wind data are shown in Fig. 7 for different integration periods, and using  $\rho_a = 1.25 \text{ kg} \cdot \text{m}^{-3}$  and  $c_d = (0.75 + 0.067 W_{10})$ . Using wind data from Poseidon the values of  $R_1$  are reduced by a factor of 2.5.

### Discussion of results

The absolute values of  $R_1$  (Fig. 7) should be taken with much precaution. Besides the uncertainty regarding the wind speed, discussed above, errors in defining the layers may cause an error in the calculated energy change, especially for small integration periods  $T$ . However, the variation of  $R_1$  with time (period of integration  $T$ ) is similar at the 3 stations and should be real. The temperature records show a maximum rate of decrease of temperature in the upper layer for the period 8 - 12 hours after the start of the storm. Considering that the inertial period is about 14 hours this might indicate the time when the rate of entrainment decreases as predicted by various theoretical models, e.g. de Szoeke and Rhines (1975).

The method of using progressively longer periods of integration (or averaging) will tend to decrease the fluctuations of  $R_1$ . Alternatively a fixed period may be used (Fig. 8), where 12 hours integration time was used throughout. The maximum again comes out for all three stations at approximately the same time as before.

The average values are 2.5, 3.3 and 4.5, all times  $10^{-3}$ , for stations 610, 611, 612, respectively, whereas the other method gave the mean values, 2.2, 3.4 and 3.2  $\cdot 10^{-3}$ , respectively. It is noticed that the long time values, 36 to 48 hours, which are the most reliable, show a relatively small scatter (Figs. 7 and 8).

Kundu (1980) found a variation with time of  $R_1$ , the maximum value being  $2.0 \cdot 10^{-3}$ , for larger times decreasing to a constant value of  $1.25 \cdot 10^{-3}$ . This yields a ratio of maximum to mean of 1.6. In the present case this ratio is 1.4 - 1.8 for the first method of integration and 1.7 - 2.4 for the second.

Other reported values of  $R_1$  are in the range  $(0.8 - 2.9) \cdot 10^{-3}$ , based on laboratory experiments as well as field studies (Kraus and Turner 1967, Kato and Phillips 1969, Denman and Miyake 1973, Kullenberg 1976).

The present  $R_1$  values show a similar time behaviour as found by Kundu (1980). The response of the sea seems in the present case to be somewhat slower than in the model case studied by Kundu. This could, however, be due to the difficulty in defining the onset time of the storm, and could also be related to the difference in initial conditions between model and reality.

Finally the value of the critical flux Richardson number based on the relation  $Rf_c = R_1/k$  is found to be in the range 0.09 - 0.18, using  $k = 2.5 \cdot 10^{-2}$  and the range of mean values of  $R_1$ . This range for  $Rf_c$  agrees quite well with what has been reported in the literature (e.g. Turner 1973, Bo Pedersen 1980).

### Acknowledgements

Financial support from the Danish Environmental Protection Agency through the use of R/V Martin Knudsen is acknowledged as well as good cooperation with the crew of the ship.

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Table 1 Aanderaa current meter measurements during BOSEX 77.

Station no.	Latitude	Longitude	Mean Depths of Current meters (m)	Depth (m)	Period of operation
610	55°50' N	18°15' E	22, 32, 43, 58, 69	78	770906/10.10am- 770920/ 2.00p.m.
611	55°51' N	18°44' E	17, 27, 37, 85, 96	109	770906/ 1.10p.m.- 770920/ 2.00p.m.
612	55°59' N	18°31' E	12, 22, 33, 80, 92	106	770906/ 3.40p.m.- 770919/12.00

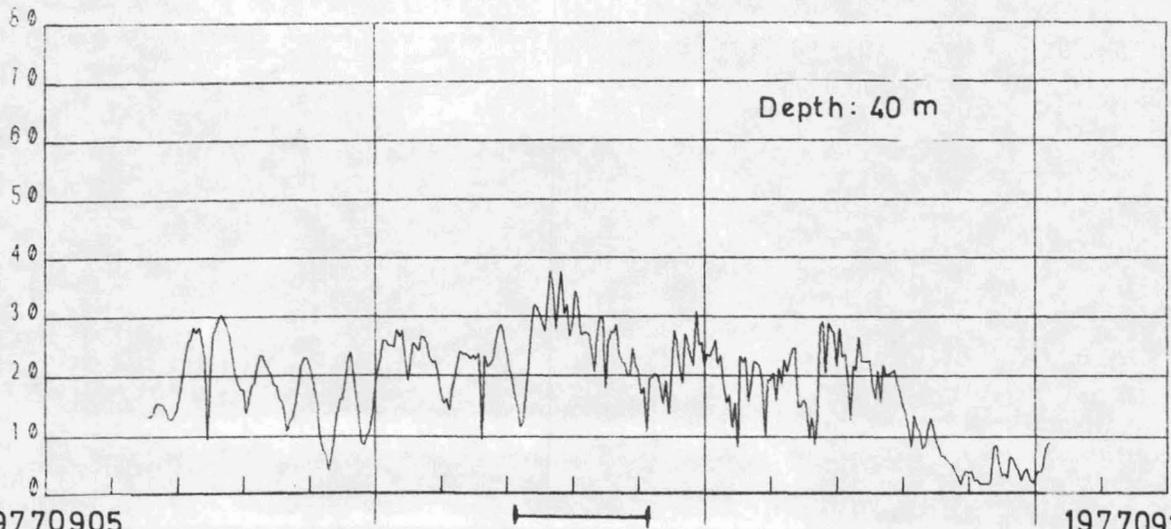
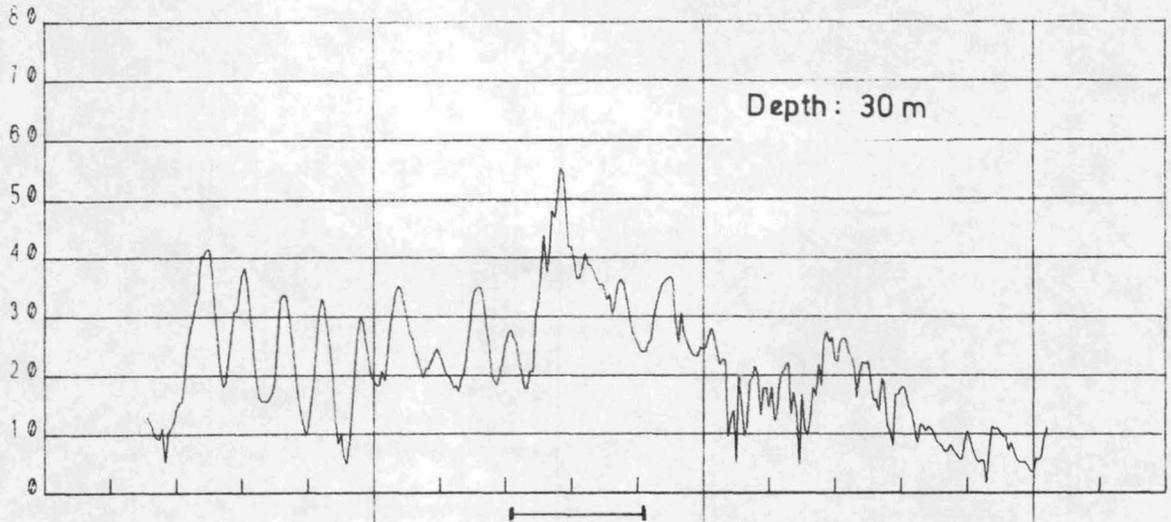
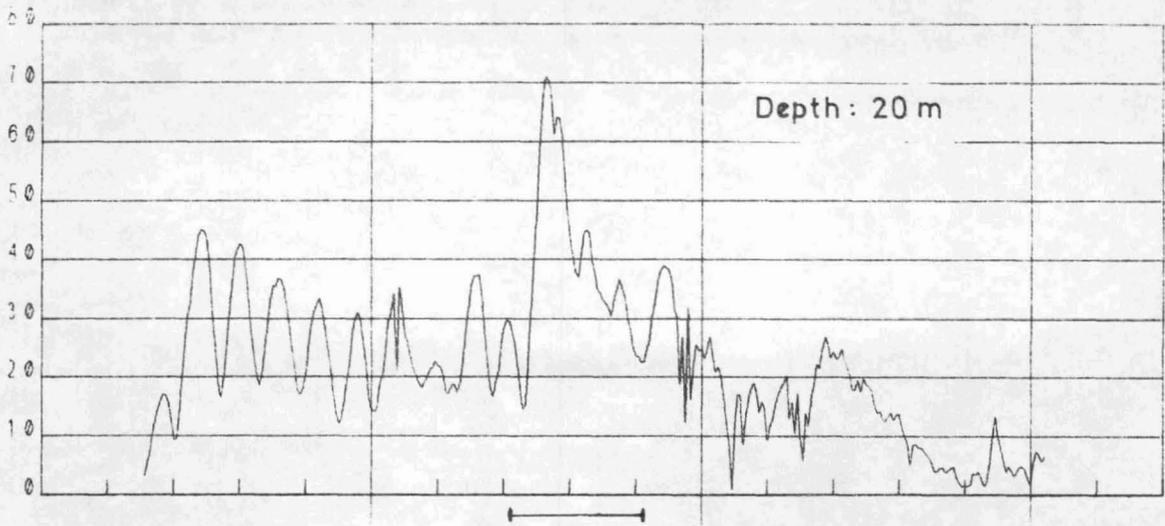
Table 2 Wind conditions during BOSEX 77  
 \* The windspeed was given in Beaufort scale  
 \*\* From Krauss 1978.

Time	R/V "ALKOR" direction	R/V "ALKOR" m/s	R/V "POSEIDON" m/s
770912/	4.00 am	SSW	6.5
	8.00 am	SW	12.5
	9.00 am		11
	12.00	W	19
	4.00p.m.	NNW	15.5
	6.00p.m.		28
	8.00p.m.	N	19
770913/	0.00	NW	12.5
	4.00 am	NW	12.5
	8.00 am	NW	14
	12.00	NW	14
	4.00p.m.	NW	14
	8.00p.m.	NW	12.5
770914/	0.00	NW	12.5
			15

BOSEX-2 South Station 611

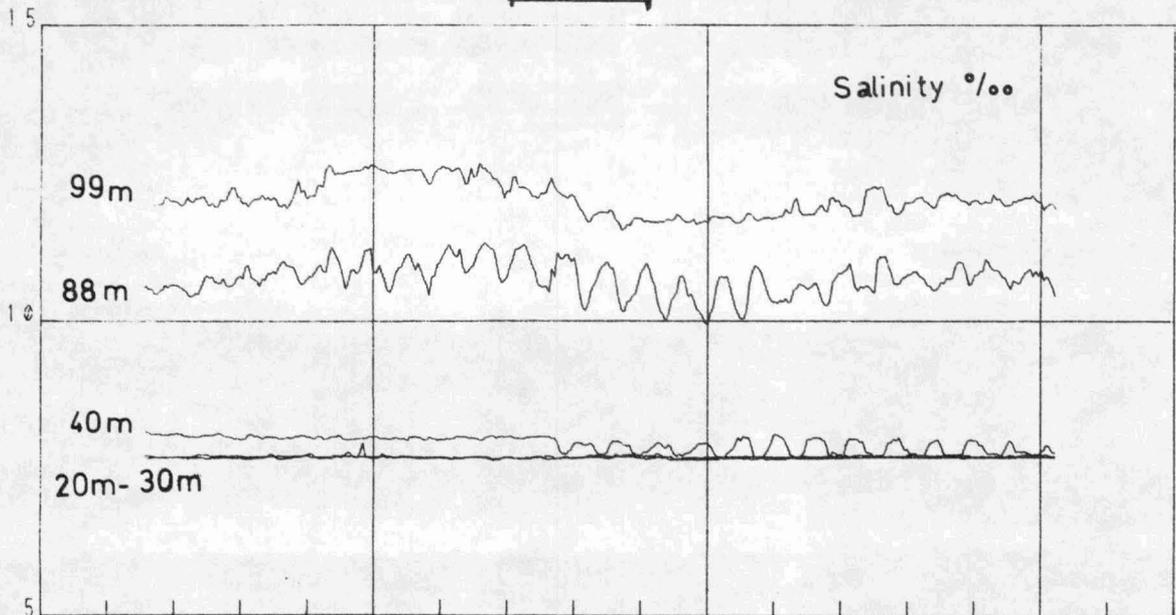
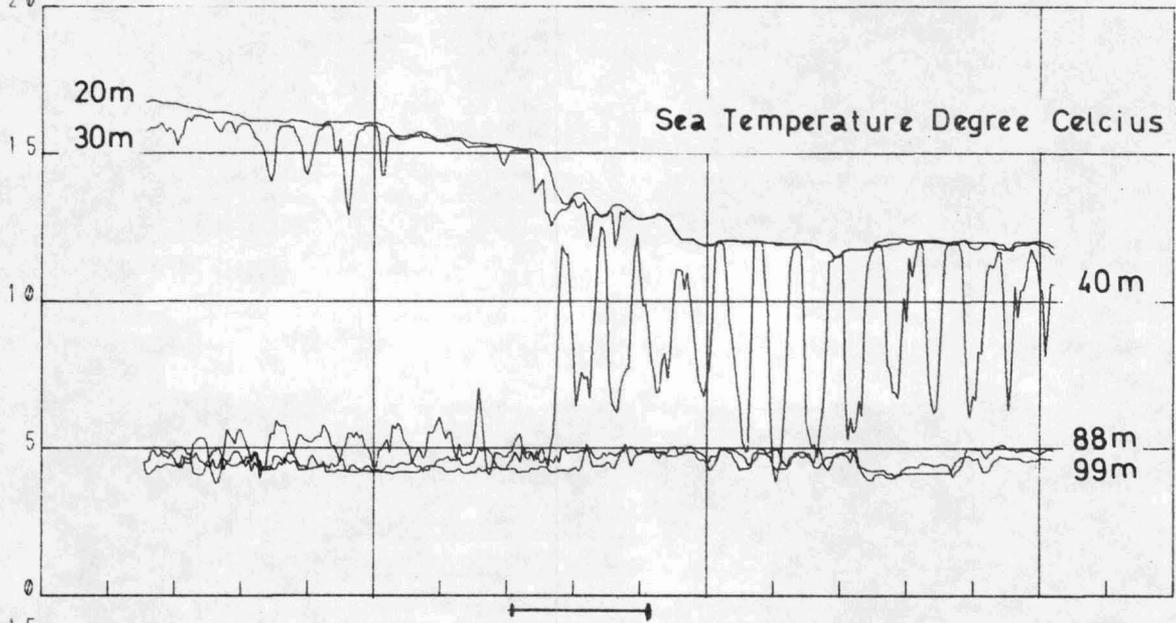
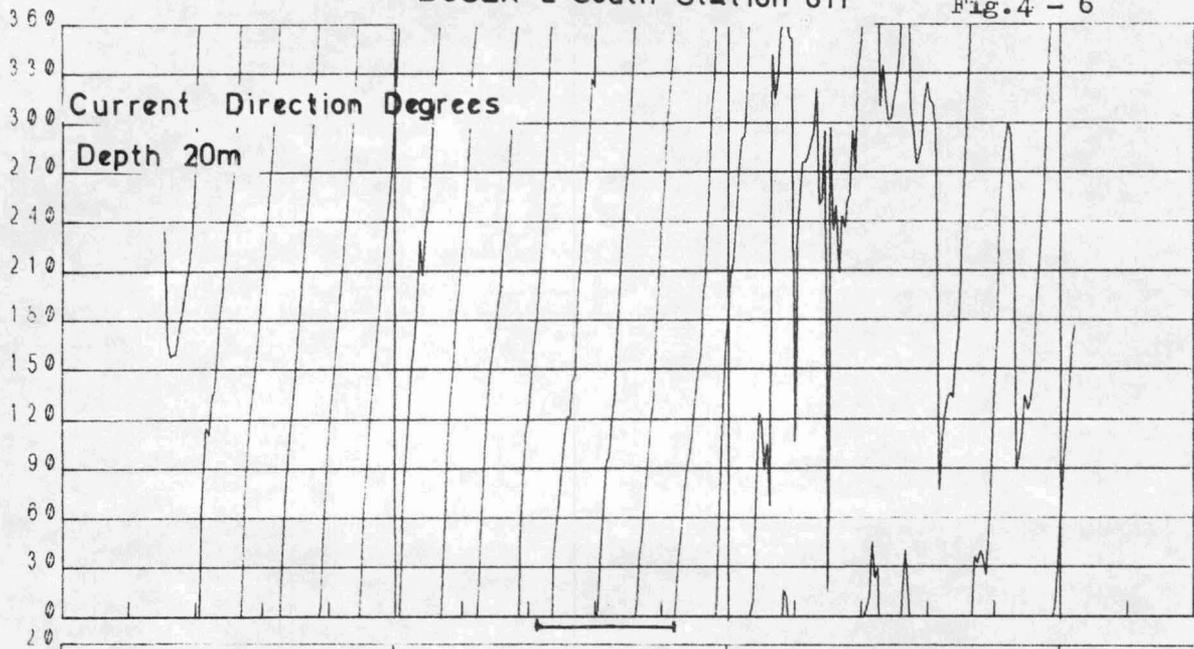
Current Speed cm/sec.

Fig. 1 - 3



19770905

19770922



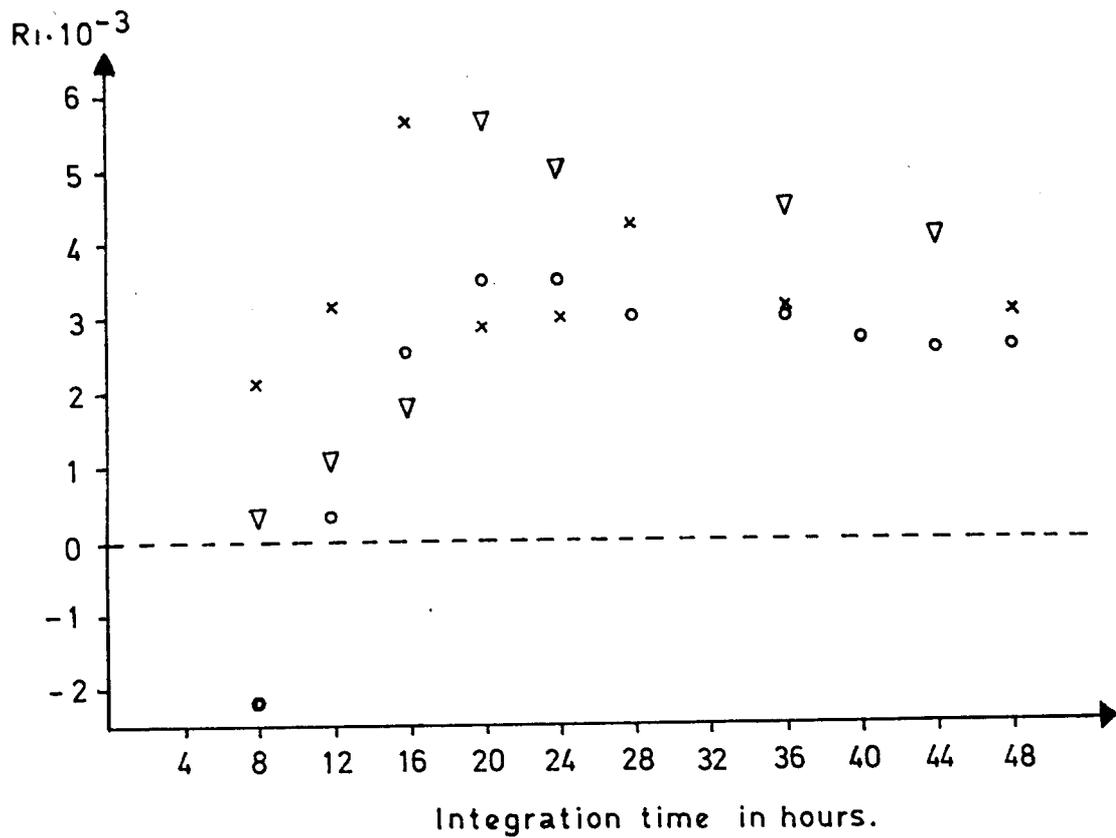


Fig. 7

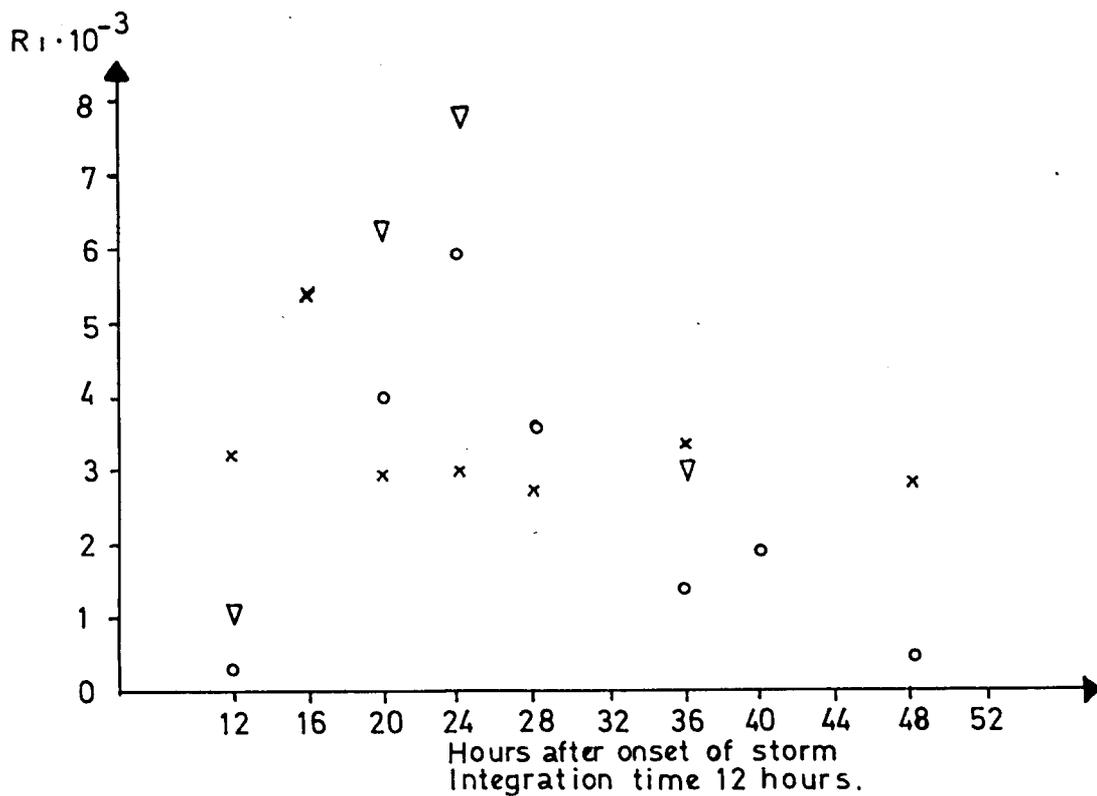


Fig. 8

○ : Station 610 ;    × : Station 611 ;    ▽ : Station 612 .